Three-dimensional theory of an ion-ripple laser

Zhi-Min Dai[1] and Lei Xu[1,2]
[1]*Institute of Nuclear Research, Academia Sinica, P.O. Box 800-204, Shanghai 201800, China**
[2]*China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China*
(Received 19 July 1993)

We develop a three-dimensional theory of the ion-ripple laser by using a consistent set of wave and pendulum equations to characterize the coherent radiation in the fundamental and harmonic frequencies. The small-signal gain is obtained by an eigenmode analysis. The optical-guiding effects on the harmonic generation in the linear regime are discussed. Under proper conditions, the ion-ripple laser may provide a practical source of microwaves, light rays, and even x rays.

PACS number(s): 41.60.Cr, 52.40.Mj, 52.75.Ms

I. INTRODUCTION

Research on tunable high-power coherent radiation sources ranging from the far infrared to ultraviolet has been a subject of growing interest. A promising candidate, the free-electron laser (FEL), has been theoretically analyzed [1,2] and experimentally demonstrated [3,4]. However, because of technological limitations of wiggler wavelengths and magnetic-field strength, the conventional FEL requires a very-high-energy (e.g., about 1 GeV) electron beam to produce short-wavelength radiation (e.g., ultraviolet), but operates there with low efficiency. Although some nonconventional FEL's [2] can provide shorter wigglers, the wiggler field is not steady or intense enough. To overcome these difficulties, Chen and Dawson recently proposed an alternative scheme [5], the ion-ripple free-electron laser (IRL), consisting of a relativistic electron beam (REB) injected into an unmagnetic performed plasma ripple at an angle θ. The IRL makes use of the charge neutralization to transport the beam and a resonance, akin to that of the planar wiggler FEL, to produce coherent radiation. Here, the wiggler field is provided by the electrostatic field of the ion ripple. Compared to conventional or nonconventional FEL's, the IRL appears attractive because its wiggler wavelength can be shorter and easily adjusted; the ion-ripple field field is steady and very strong and there is no need for an external magnetic system.

In a one-dimensional theory of the IRL [5], the linear dispersion relation for the wave coupling of the fundamental frequency for the IRL was derived from fluid theory, and was used to calculate the radiation frequency and linear growth rate. In this paper, we present a three-dimensional theory to describe the coherent radiation in the fundamental and harmonic frequencies of the IRL based on the Lorentz equation and the Maxwell equations. The main emphasis of this paper is on the role of the diffraction in the small-signal high-gain regime. In an IRL, there exist two mechanisms of optical guiding, which will be labeled as "plasma guiding" and "self-focusing." The former one, "plasma guiding," refers to the guiding of the optical beam by a waveguide, consisting of the beam channel and the surrounding plasma; here the channel has a higher dielectric constant than the surrounding plasma because of the relativistic mass increase. The latter one refers to self-similar propagation of the optical wave around a fiber with an effective complex index of refraction, because of the bunching on the optical wavelength due to the resonant coupling between the optical field and the REB [6–8]. The present paper is organized as follows: first, a general description of an IRL is given in Sec. II; in Sec. III, a consistent set of wave and pendulum equations for three-dimensional IRL's have been derived from the relativistic Lorentz equation and the Maxwell equations; in Sec. IV, an eigenmode analysis of an IRL is presented; the gain of the fundamental mode LP_{01} is analyzed in Sec. V; the higher-harmonic generation in an IRL is discussed in Sec. VI; finally, a conclusion is given in Sec. VII.

II. GENERAL DESCRIPTION OF AN ION-RIPPLE LASER

The IRL generally requires a mechanism to produce a plasma ripple. The methods for producing plasma ripple involve the modulation of the density of the neutral gas in a tank and then the ionization of the modulated gas by a laser pulse [9], or the excitation of an ion acoustic wave in an unmagnetic performed plasma [10], etc. As the beam propagates through the plasma ripple, it expels plasma electrons from the beam volume in response to the space charge. When the beam density n_b is equal to or higher than the plasma electron's density n_0, all plasma electrons are expelled from the beam path, creating an ion-ripple channel and a plasma cladding [11]. Since the time scale of interaction is much shorter than that of the ion motion, the ion ripple will be seen as a stationary wiggler by the beam electrons. As we neglect the collisions of the plasma electrons and the axial and radial (r) variation of density of the plasma cladding, the channel and the plasma cladding serve as a dielectric waveguide, which is known as a plasma waveguide [12].

*Address for correspondence.

We take the density of the ripple to be $n_i = n_0[1 + \epsilon_{\rm IR}\cos(\mathbf{k}_{\rm IR}\cdot\mathbf{r})]$, where $\epsilon_{\rm IR}$ is the perturbation ripple density, and $\mathbf{k}_{\rm IR}$ is the wave vector of the ion ripple. To reach effective higher-frequency lasing, it may be advantageous to operate at the short-wiggler-wavelength domain, which is defined as $\kappa \ll \gamma_0^{1/2}$, where γ_0 is the initial Lorentz factor of the beam electrons, and $\kappa = k_p/k_u$, with $k_u = k_{\rm IR}\cos(\theta)$ and $k_p = \omega_{pe}/c$ [c is the speed of light, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the plasma frequency with m_e and e being the rest mass and charge of the electrons, respectively]. In this case, the ion-ripple field is much stronger than the self-field on the beam, so then the electrostatic instability [13], which may be excited in the ion ripple, will be avoided. For simplicity, we neglect the effects on the self-field and the transverse $(\hat{\mathbf{x}},\hat{\mathbf{y}})$ variation of the ion-ripple field on the beam electrons. As a result, the electrostatic field experienced by the beam electrons can be expressed by

$$\mathbf{E}_{\rm IR} = -\frac{4\pi n_0 e}{k_u}\epsilon_i \sin(k_u z)[\hat{\mathbf{x}}\sin(\theta) - \hat{\mathbf{z}}\cos(\theta)] , \tag{2.1}$$

where $\epsilon_i = \epsilon_{\rm IR}\cos(\theta)$ is the fractional ripple of ion density (we have assumed that the beam density is equal to or higher than the average plasma density). To avoid the "ion-hose" instability [14] and the "electron-hose" instability [15], we will keep the average plasma density nearly the same as the unperturbed density of the REB (i.e., $n_0 \approx n_b$).

The equilibrium trajectories of the REB moving in the ion-ripple field are determined by the Lorentz equation. By using Eq. (2.1), one obtains the following, with the assumption that the slow time scale solutions are approximately constant on the fast time scales

$$\beta_x = -\frac{\sqrt{2}K}{\gamma_0}\cos(k_u \bar{z}) , \tag{2.2}$$

$$\beta_z = \bar{\beta}_z - \frac{K^2}{2\gamma_0^2}\cos(2k_u\bar{z}) , \tag{2.3}$$

where $\bar{z}(t) = \bar{\beta}_z ct$ is the axial location assuming the electron has uniform axial velocity $\bar{\beta}_z = 1 - 1/2\gamma_0^2(1+K^2)$, and $K = \kappa^2\epsilon_i\sin(\theta)/\sqrt{2}$ represents the wiggler strength for the IRL. These expressions for $\mathbf{v}$ are valid to order $(K/\gamma_0)^2$. It should be noted that the transverse fast oscillations given in Eq. (2.2) are the source of the energy needed to produce the electromagnetic radiation [16], and the additional longitudinal $(\hat{\mathbf{z}})$ fluctuations $(2k_u)$ in Eq. (2.3) may cause emission into higher harmonics and reduce the gain of the fundamental [17].

III. BASIC EQUATIONS OF AN ION-RIPPLE LASER

In the presence of a REB, the ion-ripple field couples to a high-frequency radiation wave. The vector potential of the linearly polarized radiation field is taken to be

$$\mathbf{A} = \frac{m_e c^2}{2ke}E_s \exp[i(kz - \omega t)]\hat{\mathbf{e}}_x + \text{c.c.} , \tag{3.1}$$

where E_s is the complex radiation amplitude, and ω and k are respectively the frequency and axial wave number of the radiation. The radiation fields satisfy the wave equation

$$\left[\nabla^2 - \frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{A} = -\frac{4\pi}{c}\mathbf{J} , \tag{3.2}$$

where $\mathbf{J}$ is the driving current density associated with the medium, and ϵ denotes the partial dielectric constant, which is given by

$$\epsilon = 1 - u\omega_{pe}^2/\omega^2\gamma_0 - \tilde{u}\omega_{pe}^2/\omega^2 , \tag{3.3}$$

with the step function $u = 1$ for $r < r_0$, $u = 0$ for $r > r_0$, and $\tilde{u} = 1 - u$; here r_0 is the channel (or beam) radius. Usually, the paraxial approximation is employed for the analysis of the coherent emission from FEL's [18]. Within the confine of this approximation, and employing the calculational technique, which was exploited earlier to evaluate the gain in higher-FEL harmonics [17], Eq. (3.2) may be reduced to

$$\left[\nabla_\perp^2 - k_p^2\tilde{u} + 2ik\frac{d}{dz}\right]E_s = \frac{\sqrt{2}kk_p^2K}{\gamma_0}u\,[JJ]\langle\exp(-i\xi_n)\rangle , \tag{3.4}$$

where $\xi_n = (k + nk_u)\bar{z} - \omega t$ is the ponderomotive phase for the nth harmonic frequency with the harmonic number $n = 1,3,5,\ldots$. The symbol $\langle\ \rangle$ indicates an ensemble average in the electron's longitudinal phase space. The factor $[JJ]$ clearly represents the effects of the longitudinal fast oscillation on the stimulated emission in an IRL as well as that in a FEL, which is defined as

$$[JJ] = (-1)^{(n-1)/2}[J_{(n-1)/2}(n\sigma) - J_{(n+1)/2}(n\sigma)] , \tag{3.5}$$

where $\sigma = K^2/2(1+K^2)$ denotes the interaction strength of the IRL, and J_n is the Bessel function of the first kind and nth order. Introducing dimensionless parameters $\tau = k_u\bar{z}$, $\tilde{r} = \sqrt{2kk_u}\,r$, and $\tilde{k}_p = k_p/\sqrt{2kk_u}$, Eq. (3.4) may be rewritten as

$$\left[\tilde{\nabla}_\perp^2 - \tilde{k}_p^2\tilde{u} + i\frac{d}{d\tau}\right]E_s = \frac{Kk_p^2}{\sqrt{2}\gamma_0 k_u}u\,[JJ]\langle\exp(-i\xi_n)\rangle . \tag{3.6}$$

When the number of the ion-ripple periods $N \gg 1$, the evolution of the ponderomotive phase is determined by

$$\frac{d^2}{d\tau^2}\xi_n = \frac{d}{d\tau}\nu_n = \frac{2n}{\gamma_0}\frac{d}{d\tau}\gamma , \tag{3.7}$$

where ν_n represents the energy detuning for the nth harmonic frequency. In Eq. (3.7), the change of electron energy with time is governed by the Lorentz equation

$$\frac{d}{dt}\gamma = -\frac{e}{m_e c}\boldsymbol{\beta}\cdot\mathbf{E} , \tag{3.8}$$

where $\mathbf{E}$ is the electromagnetic vector with the following relation between the field and the potential $\mathbf{E} = -\partial A/c\partial t$. On using Eqs. (3.1), (3.7), and (3.8), we could get the pendulum equation after performing averages over one or several optical wavelengths

$$\frac{d^2}{d\tau^2}\xi_n = i\frac{KE_s}{\sqrt{2}\gamma_0^2 k_u} n[JJ]\exp(i\xi_n) + \text{c.c.} \tag{3.9}$$

Together with Eqs. (3.6) and (3.9), the self-consistent wave and electron equations of motion yield a powerful formulation for the three-dimensional IRL problem, which are valid in weak and strong optical fields, as well as for small or high gain. If we neglect the effects of the plasma waveguide in an IRL, these equations are similar with the well-known FEL results [6–8].

IV. LINEAR ANALYSIS OF AN ION-RIPPLE LASER

We now proceed to solve Eqs. (3.6) and (3.9) in the small-signal regime. Under small-signal conditions, the reference to the individual electron phase can be explicitly removed by combining Eqs. (3.6) and (3.9). Then, one obtains

$$\left[\tilde{\nabla}_\perp^2 - \tilde{k}_p^2 \tilde{u} + \nu_n + i\frac{d}{d\tau}\right]\tilde{E}_s = u j_n \tilde{I} , \tag{4.1}$$

where $\tilde{E}_s$ and $\tilde{I}$ are defined as

$$\tilde{E}_s = E_s \exp(i\nu_n\tau), \quad \tilde{I} = \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' \tilde{E}_s(\tau'') , \tag{4.2}$$

with the initial condition $\tilde{I}(\tau=0)=0$, and $\partial\tilde{I}/\partial\tau(\tau=0)=0$, and the dimensionless current j_n is given by

$$j_n = \frac{\kappa^6 \epsilon_i^2 \sin^2(\theta)}{4\gamma_0^3} n[JJ]^2 . \tag{4.3}$$

Following Xie, Deacon, and Madey [6], one can rewrite Eq. (4.1) as a Schrödinger equation

$$i\frac{d}{d\tau}\Psi = H\Psi , \tag{4.4}$$

with non-Hermitian Hamiltonian

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -L_1 & 1 \\ L_2 & 0 & 0 \end{bmatrix} , \tag{4.5}$$

and the vector solution

$$\Psi = \begin{bmatrix} -id\tilde{I}/d\tau \\ \tilde{E}_s \\ -L_2\tilde{I} \end{bmatrix} , \tag{4.6}$$

where $L_1 = [\tilde{\nabla}_\perp^2 - \tilde{k}_p^2 \tilde{u}] + \nu_n$ and $L_2 = u j_n$. Equation (4.4) is the governing equation for the evolution of both the optical field and the modulation of the REB. It defines a linear eigenvalue problem by $HV = \lambda V$. Since the function u and $\tilde{u}$ depend only on the transverse spatial (r), the solution of Eq. (4.4) is self-similar [7]. The term "self-similar" refers to the fact that the transverse dependence of the mode is independent of the axial coordinate z. In the high-gain regime, the fast growing mode $\Psi(\tilde{r}) = V_l \exp(-i\lambda_l\tau)$, $\text{Im}(\lambda_l) > 0$ grows to dominate over all other modes, where V_l is defined as

$$V_l = \psi_l \begin{bmatrix} \lambda_l^{-1} \\ 1 \\ 2u j_n \lambda_l^{-2} \end{bmatrix} , \tag{4.7}$$

where λ_l is the generally complex propagation constant, and the transverse profile ψ_l obeys the mode equation

$$[\tilde{\nabla}_\perp^2 - \tilde{k}_p^2 \tilde{u} + \lambda_l + \nu_n - u j_n/\lambda_l^2]\psi_l = 0 . \tag{4.8}$$

The evolution of the coupling system can be solved as an initial problem defined by Eq. (4.4) along with the initial condition. Because the Hamiltonian H in Eq. (4.5) is not Hermitian, the eigenmodes of Eq. (4.8) are not orthogonal to each other. However, according to the biorthogonality theorem [19], the eigenfunctions V_l of H are orthogonal to the eigenfunctions $V_l^\dagger$ of the adjoint operator $H^\dagger$, which is

$$\langle V_l^\dagger V_m \rangle = N_l \delta_{l,m}, \quad N_l = \langle [1 + 2u j_n \lambda_l^{-3}]\psi_l^2 \rangle . \tag{4.9}$$

Therefore, a general solution Ψ to the initial value problem can be expressed as a linear superposition of all self-similar modes. The vector Ψ has three components which are known as the velocity bunching amplitude, the optical field amplitude, and the density bunching amplitude, respectively. Here, we focus our attention on the optical component, which describes the aspects of an IRL operation associated with energy exchange. Thus, the optical field may be expressed by

$$\tilde{E}_s = \sum_l \frac{1}{N_l} \langle \tilde{E}_s(0)\psi_l \rangle \psi_l \exp(-i\lambda_l\tau) . \tag{4.10}$$

The gain of an IRL may be obtained as $G = G_0 \exp[2\,\text{Im}(\lambda_l)\tau]$, and the input power coupling coefficient G_0 is given by [6] $G_0 = |\langle \tilde{E}_s(0)\psi_l \rangle|^2 \langle |\tilde{E}_s(0)|^2 \rangle$. It shows that the power coupling G_0 depends on the input optical field as well as the mode of the transverse profile. While the eigenvalues λ_l for a given system are fixed, the power coupling G_0 varies about the profile of the input optical field. According to previous work [6,7], the maximal value of G_0 is reached with $\tilde{E}_s(0) = \psi_l^*$, which is $G_0 = \langle |\psi_l|^2 \rangle^2 / |N_l|^2$.

V. GAIN OF THE SELF-SIMILAR MODE

Since $u(r)$ is a sharp-edged distribution of radius r_0, the eigenfunction of Eq. (4.8) may be expressed in terms of Bessel functions and Hankel functions. We consider the axial symmetric guided mode, for which

$$\psi(\tilde{r}) = \begin{cases} J_0(\chi\tilde{r}/a) & \text{for } \tilde{r} < a , \\ DH_0(\chi_p\tilde{r}/a) & \text{for } \tilde{r} > a , \end{cases} \tag{5.1}$$

where the constant $D = J_0(\chi)/H_0(\chi_p)$, $a = \sqrt{2kk_u}\, r_0$, and H_0 is the Hankel function of the first kind and zeroth order. The complex parameters χ and χ_p are determined by the equations

$$\frac{\chi J_0'(\chi)}{J_0(\chi)} = \frac{\chi_p H_0'(\chi_p)}{H_0(\chi_p)} \equiv \eta , \tag{5.2}$$

$$\chi^2 - \chi_p^2 = \hat{b} - \Gamma_n \hat{b}^3 (\chi_p^2 + \hat{b} - \hat{\nu}_n)^{-2} \equiv V^2 , \tag{5.3}$$

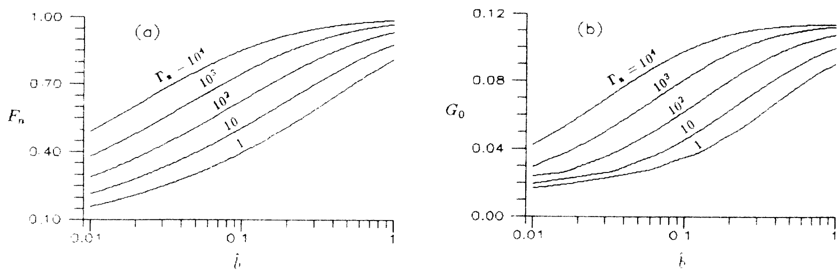

FIG. 1. Filling factor F_n (a), and the power coupling coefficient G_0 (b) are plotted as a function of $\hat{b}$ for different Γ_n, $\Gamma_n = 1$, 10, 10^2, 10^3, 10^4.

with

$$\Gamma_n = \frac{16\gamma_0^3 \epsilon_i^2 \sin^2(\theta)}{(1+K^2)^3} n^4 [JJ]^2 . \tag{5.4}$$

The parameters $\hat{\nu}_n = a^2 \nu_n$, and $\hat{b} = k_p^2 r_0^2 \simeq 4(I_b / I_A)$, with I_b being the beam current and $I_A = m_e c^3/e \simeq 17$ kA being the Alfvén current.

It is worthwhile, at this point, to examine some important features of the eigenmode by contracting it with the LP_0 mode in a fiber [20]. Obviously, the eigenmode described by Eqs. (5.1)–(5.3) is similar to the LP_0 mode in fiber, where the quantity V is called the "fiber parameter." The condition for mode cutoff in a fiber is $\chi_p \rightarrow 0$. Formally, there is no cutoff for the LP_{01} mode corresponding to the first solution of Eqs. (5.2) and (5.3), which is expected to be the dominant mode for an IRL.

The growth rate could be expressed by

$$g_n = \mathrm{Im}(\lambda_n) = \frac{\sqrt{3}}{2} F_n j_n^{1/3} , \tag{5.5}$$

with

$$F_n = 2\,\mathrm{Im}(\chi_p^2)/\sqrt{3}\Gamma_n^{1/3}\hat{b} . \tag{5.6}$$

The factor F_n clearly represents the effects of the optical guiding on the growth rate, which may be understood as a filling factor in the IRL. With the choice of initial field $\tilde{E}(0) \simeq \psi^*$, the input power coupling coefficient G_0 may be obtained as [7]

$$G_0 = T_0^2 / |N_0|^2 , \tag{5.7}$$

where

$$T_0 = a^2 |J_0(\chi)|^2 (\eta^* - \eta) \left| \frac{1}{(\chi^2 - \chi^{*2})} - \frac{1}{(\chi_p^2 - \chi_p^{*2})} \right| , \tag{5.8}$$

$$N_0 = \frac{a^2}{2} J_1^2(\chi) \left| 2\Gamma_n \hat{b}^3 (\chi_p^2 + \hat{b} - \hat{\nu}_n)^{-3} \left| 1 + \frac{\chi^2}{\eta^2} \right| + 1 - \frac{\chi^2}{\chi_p^2} \right| . \tag{5.9}$$

It shows that the filling factor F_n and the power coupling G_0 are dependent upon the parameters Γ_n and $\hat{b}$. The effects of the optical guiding on the gain of the coherent radiation are better clarified by Fig. 1, in which we compare the factor F_n and the initial power coupling G_0 versus $\hat{b}$ for different Γ_n with the effective detuning $\hat{\nu}_n$ being chosen to the maximize F_n. Our numerical results turn out that the maximum gain occurs for a positive value of the energy detuning. As the energy detuning is smaller than the radiation frequency, the resonant frequency is approximately given by

$$\omega = 2n\gamma_0^2 k_{\mathrm{IR}} c \cos(\theta)/(1+K^2) , \tag{5.10}$$

for the nth harmonic.

Besides the transverse localization of the optical beam extent, the plasma waveguide diminishes any convective loss of radiation energy and enhances the gain of the IRL. In the limit of $\Gamma_n \gg 1$ and low beam current $\hat{b} \ll 1$, the effects of the plasma waveguide may be negligible, and the "self-focusing" dominate the optical guiding. In this case, Eq. (5.3) will be reduced to

$$\chi^2 = \chi_p^2 - \hat{C}_n (\chi_p^2 - \hat{\nu}_n)^{-2} , \tag{5.11}$$

with $\hat{C}_n = \Gamma_n \hat{b}^3$, which is similar to the FEL form discussed by Moore [7]. The transverse profile of the self-similar optical field is governed by $\hat{C}_n$. For the LP_{01} mode to be considered guided, we require the $1/e$ point of the electromagnetic wave to be within 5 times the channel radius a. This condition corresponds to demanding that $|\mathrm{Re}(V^2)| + |\mathrm{Im}(V^2)| > 2.4$ [8].

As $\hat{C}_n$ is proportional to $(\gamma_0 I_b)^3$, the transverse profile ψ_l, the filling factor F_n and the power coupling coefficient G_0 depend on $\gamma_0 I_b$ rather than the beam current I_b. Different from the FEL problem, the gain of the IRL is independent of the radius of the beam r_0, as the gain of the FEL is sensitively dependent upon r_0 [6–8]. In Fig. 2, we plot the filling factor F_n and the power coupling coefficient G_0 versus $\hat{C}_n$ where the effective detuning $\hat{\nu}_n$ is chosen to maximize F_n. For small $\hat{C}_n$, the filling factor F_n and the power coupling G_0 are much smaller than the

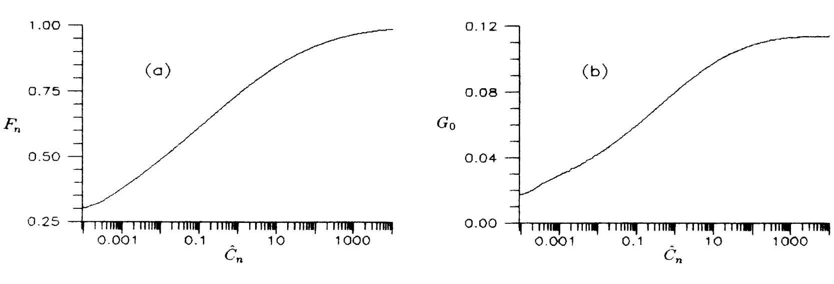

FIG. 2. Filling factor F_n (a), and the power coupling coefficient G_0 (b) are shown as a function of the parameter $\hat{C}_n$ in the "self-focusing" domain.

one-dimensional results, since radiated light would rapidly diffract out of the electron beam before it could become very amplified. For large $\hat{C}_n$ (e.g., $\hat{C}_n \geq 10^2$), the diffractive effects may be relatively unimportant, and the optical field is strongly trapped in the beam volume. Therefore, the filling factor F_n and the power coupling G_0 approach the one-dimensional value 1 and $\frac{1}{9}$, respectively.

VI. HIGHER-HARMONIC GENERATION IN AN ION-RIPPLE LASER

In Sec. V, we have studied the gain of the IRL in the small-signal high-gain regime, and our analysis shows that the growth rate g_n is proportional to the dimensionless current $j_n^{1/3}$ and the filling factor F_n. It can be seen from Eq. (4.3) that it requires a higher κ (i.e., $\omega_{pe} \gg k_u c$) to obtain higher gain for a fixed REB. This is something of a disadvantage. As $j_n \sim n[JJ]^2$, there exists a lot of harmonic emission for $\kappa \gg 1$. Thus, to reach a higher frequency with high gain, it may be advantageous to operate at higher harmonics.

However, harmonic operation has often been limited by problems of mode competition, insufficient gain, and low efficiency, which get worse with increased harmonic number. Thus, harmonic IRL operation generally requires a mechanism that selectively enhances the competitiveness of a desired harmonic, such as is done for the FEL. A few methods for harmonic selection have been proposed in FEL operation. For example, resonantor tuning [4], wiggler-field tapering [21], signal injection [22], periodic positioning interaction [23], etc. These methods may be employed to make a realizable harmonic operation in an IRL.

In this paper, we propose an alternative mechanism—the diffraction loss to suppress the gain of the fundamental and the lower harmonics, making it possible to have higher-harmonic operation. In the "self-focusing" domain, the filling factor F_n and the power coupling G_0 are only governed by the parameter $\hat{C}_n$, which is proportional to $n^4[JJ]^2$. It indicates that F_n and G_0 are both different for different harmonics. Under proper conditions, the optical field of the higher harmonics may be strongly localized, resulting in a higher gain, but the optical wave of the fundamental and the lower harmonics extend far outside of the beam leading to a much lower gain. The effects of the optical guiding on higher-harmonic generation is better clarified by Fig. 3, in which we compare the F_n, G_0, and g_n as a function of $n/(1+K^2)$ for $K=10,15,20,25,30$ together with their magnitude with $\gamma_0=1000$, $\hat{b}=0.01$, $\epsilon_i \sin(\theta)=0.1$. Obviously, the growth rate of the fundamental and lower harmonics may be suppressed by diffraction loss and then the gain of higher harmonics may be higher. This gives a possibility of higher-harmonic operation.

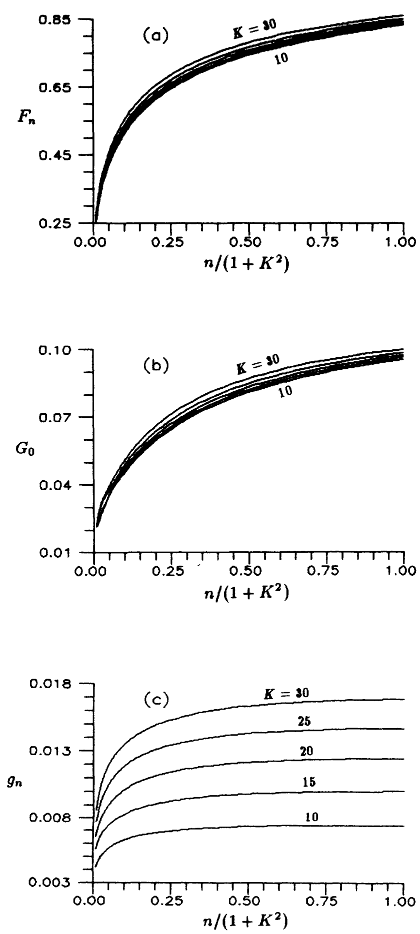

FIG. 3. Filling factor F_n (a), the power coupling coefficient G_0 (b), and the growth rate g_n are plotted as a function of $n/(1+K^2)$ for $K=10, 15, 20, 25, 30$.

VII. CONCLUSION

In this paper, we presented a three-dimensional theory of an ion-ripple laser. Based on the Lorentz equation and the Maxwell equations, we developed a consistent set of wave and pendulum equations to describe the coherent radiation in fundamental and odd harmonic frequency in an IRL. The gain of the self-similar mode for an IRL was obtained by an eigenmode analysis in the weak-field and high-gain regimes. Our results show that the emission frequency is peaked at $\omega=2n\gamma_0^2 k_{\mathrm{IR}} c\cos(\theta)/(1+K^2)$ for the nth harmonic. Moreover, we discussed the potential of higher-harmonic operation in an IRL. It is found that the effects of the diffraction loss could reduce the gain of the fundamental and lower harmonics, and make possible higher-harmonic operation.

The optical-guiding effects are dependent upon the parameters Γ_n and $\hat{b}$. In the limit of $\Gamma_n \gg 1$ and low current $\hat{b} \ll 1$, the "self-focusing" dominates the optical guiding. In this case, a dimensionless parameter $\hat{C}_n$ is introduced to determine whether diffraction is important. For small $\hat{C}_n$, the diffraction loss is important, and the gain is much smaller than that of the one-dimensional results. For $\hat{C}_n > 10^2$, the diffraction loss is negligible, and the gain approximately equal to the one-dimensional value.

Because of the effects of the optical guiding, it is possible to direct and focus the IRL-generated optical beam.

Furthermore, since the ion ripple can be created with very short wavelength, and with the potential of higher-harmonic operation, we expect that the IRL is a realistic means for generating short-wavelength tunable coherent radiation. It would appear to be possible to produce tunable uv and soft x rays from an IRL.

In conclusion, to take account of the optical-guiding effects in our analysis we have assumed that the electron transverse oscillation scale is smaller than the radial length structure (i.e., $\kappa^3\epsilon_i\sin(\theta) \ll \gamma_0\hat{b}^{1/2}$), so that the effect of the REB bending due to the finite radius of the electron wiggling may be neglected. Adding the wiggling motion will decrease gain and increase transverse dimensions [24]. However, these effects are significant only for $\kappa^3\epsilon_i\sin(\theta) \gg \gamma_0\hat{b}^{1/2}$. Therefore, the smooth boundary approximation in our analysis is valid enough.

[1] J. M. J. Madey, J. Appl. Phys. **42**, 1906 (1971); D. Prosnitz, A. Szoke, and V. K. Neil, Phys. Rev. A **24**, 1436 (1981).

[2] L. R. Elias, Phys. Rev. Lett. **42**, 977 (1979); G. Bekefi, J. S. Wurtele, and I. H. Deutsch, Phys. Rev. A **34**, 1228 (1986).

[3] D. A. G. Deacon, L. R. Elias, J. M. J. Madey, G. J. Ramian, H. A. Schwettman, and T. I. Smith, Phys. Rev. Lett. **38**, 892 (1977); T. J. Orzechowski, B. Anderson, W. M. Fawley, D. Prosnitz, E. T. Scharlemann, S. Yarema, D. Hopkins, A. C. Paul, A. M. Sesseler, and J. Wurtele, Phys. Rev. Lett. **54**, 889 (1985); L. Vintro, S. V. Benson, A. Bhowmik, M. S. Curtin, J. M. J. Madey, W. A. Mcmullin, and B. A. Richman, Phys. Rev. Lett. **64**, 1662 (1990).

[4] S. V. Benson and J. M. J. Madey, Phys. Rev. A **39** 1579 (1989).

[5] K. R. Chen and J. M. Dawson, Phys. Rev. Lett. **68**, 29 (1992); K. R. Chen and J. M. Dawson, Phys. Rev. A **45**, 4077 (1992).

[6] M. Xie, D. A. G. Deacon, and J. M. J. Madey, Phys. Rev. A **41**, 1662 (1990).

[7] G. T. Moore, Opt. Commun. **52**, 46 (1984); G. T. Moore, Nucl. Instrum. Methods A **250**, 381 (1986).

[8] E. T. Scharlemann, A. M. Sessler, and J. S. Wurtele, Phys. Rev. Lett. **54**, 1925 (1985).

[9] A. E. Dangor, A. K. L. Dymoke-Bradshaw, A. Dyson, T. Garvey, I. Mitchell, A. J. Cole, C. N. Danson, C. B. Edwards, and R. G. Evans, IEEE Trans. Plasma Sci. **PS 15**, 161 (1987).

[10] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973), pp. 108 and 109.

[11] J. J. Su, T. Katsouleas, J. M. Dawson, and R. Fedele, Phys. Rev. A **41**, 3321 (1991).

[12] H. M. Shen, J. Appl. Phys. **69**, 6827 (1991).

[13] A. T. Lin, P. K. Kaw, and J. M. Dawson, Phys. Rev. A **8**, 2618 (1973).

[14] H. L. Buchanan, Phys. Fluids **30**, 221 (1987).

[15] D. H. Whittum, W. M. Sharp, S. S. Yu, M. Lampe, and G. Joyce, Phys. Rev. Lett. **67**, 991 (1991).

[16] K. R. Chen, J. M. Dawson, and A. T. Lin, Phys. Fluids B **3**, 1270 (1991).

[17] W. B. Colson, Phys. Rev. A **24**, 1399 (1981); W. B. Colson, IEEE J. Quantum Electron. QE **17**, 1417 (1981).

[18] M. Sargent, M. O. Scully, and W. E. Lamb, *Laser Physics* (Addison-Wesley, Reading, MA, 1974), p. 100.

[19] Bernard Friedman, *Principles and Techniques of Applied Mathematics* (Wiley, New York, 1956).

[20] D. Marcuse, *Theory of Dielectric Waveguides* (Academic, New York, 1974).

[21] H. P. Freund, Phys. Rev. A **37**, 3371 (1988).

[22] P. E. Latham, B. Levush, T. M. Antonsen, Jr., and N. Metzler, Phys. Rev. Lett. **66**, 1442 (1991).

[23] H. Bluem, R. H. Jackson, H. P. Freund, D. E. Pershing, and V. L. Granatsein, Phys. Rev. Lett. **67**, 824 (1991).

[24] R. H. Pantell, E. Fontana, and J. Feinstein, Nucl. Instrum. Methods A **259**, 143 (1987).